Paper Reference(s)

6679

Edexcel GCE

Mechanics M3

Advanced Level

Thursday 29 January 2009 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M3), the paper reference (6679), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

N29495A

1. A particle P of mass 3 kg is moving in a straight line. At time t seconds, $0 \le t \le 4$, the only force acting on P is a resistance to motion of magnitude $\left(9 + \dfrac{15}{(t+1)^2}\right)$N. At time t seconds the velocity of P is v m s^{-1}. When $t = 4$, $v = 0$.

Find the value of v when $t = 0$.

(7)

2.

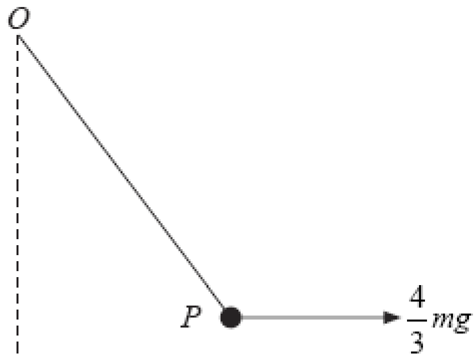

Figure 1

A particle P of mass m is attached to one end of a light elastic string, of natural length a and modulus of elasticity $3mg$. The other end of the string is attached to a fixed point O.

The particle P is held in equilibrium by a horizontal force of magnitude $\frac{4}{3}mg$ applied to P.

This force acts in the vertical plane containing the string, as shown in Figure 1. Find

(*a*) the tension in the string,

(5)

(*b*) the elastic energy stored in the string.

(4)

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3. A rough disc rotates about its centre in a horizontal plane with constant angular speed 80 revolutions per minute. A particle P lies on the disc at a distance 8 cm from the centre of the disc. The coefficient of friction between P and the disc is μ. Given that P remains at rest relative to the disc, find the least possible value of μ.

(7)

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4. A small shellfish is attached to a wall in a harbour. The rise and fall of the water level is modelled as simple harmonic motion and the shellfish as a particle. On a particular day the minimum depth of water occurs at 10 00 hours and the next time that this minimum depth occurs is at 22 30 hours. The shellfish is fixed in a position 5 m above the level of the minimum depth of the water and 11 m below the level of the maximum depth of the water. Find

(*a*) the speed, in metres per hour, at which the water level is rising when it reaches the shellfish,

(7)

(*b*) the earliest time after 10 00 hours on this day at which the water reaches the shellfish.

(4)

5.

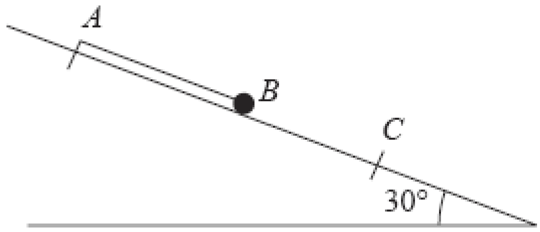


Figure 2

One end A of a light elastic string, of natural length a and modulus of elasticity $6mg$, is fixed at a point on a smooth plane inclined at 30° to the horizontal. A small ball B of mass m is attached to the other end of the string. Initially B is held at rest with the string lying along a line of greatest slope of the plane, with B below A and $AB = a$. The ball is released and comes to instantaneous rest at a point C on the plane, as shown in Figure 2.

Find

(*a*) the length AC,

(5)

(*b*) the greatest speed attained by B as it moves from its initial position to C.

(7)

6.

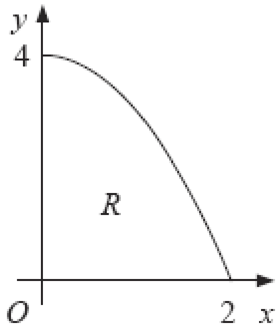


Figure 3

The region R is bounded by part of the curve with equation $y = 4 - x^2$, the positive x-axis and the positive y-axis, as shown in Figure 3. The unit of length on both axes is one metre. A uniform solid S is formed by rotating R through 360° about the x-axis.

(a) Show that the centre of mass of S is $\frac{5}{8}$ m from O.

(10)

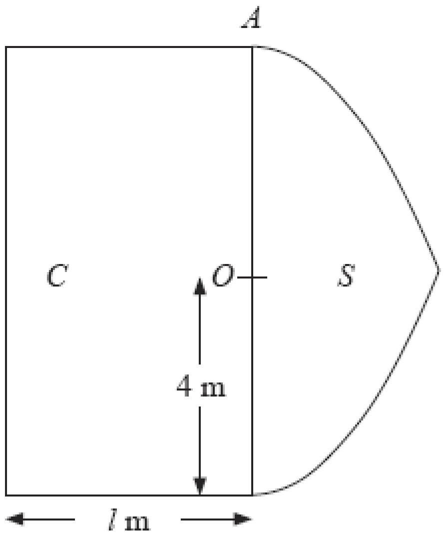


Figure 4

Figure 4 shows a cross section of a uniform solid P consisting of two components, a solid cylinder C and the solid S. The cylinder C has radius 4 m and length l metres. One end of C coincides with the plane circular face of S. The point A is on the circumference of the circular face common to C and S. When the solid P is freely suspended from A, the solid P hangs with its axis of symmetry horizontal.

(b) Find the value of l.

(4)

7.

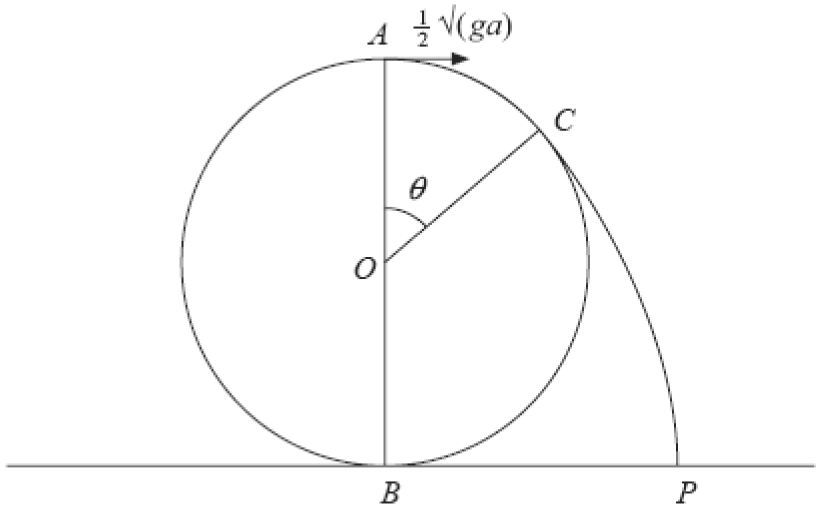


Figure 5

A particle is projected from the highest point A on the outer surface of a fixed smooth sphere of radius a and centre O. The lowest point B of the sphere is fixed to a horizontal plane. The particle is projected horizontally from A with speed $\frac{1}{2}\sqrt{(ga)}$. The particle leaves the surface of the sphere at the point C, where $\angle AOC = \theta$, and strikes the plane at the point P, as shown in Figure 5.

(a) Show that $\cos\theta = \frac{3}{4}$.

(7)

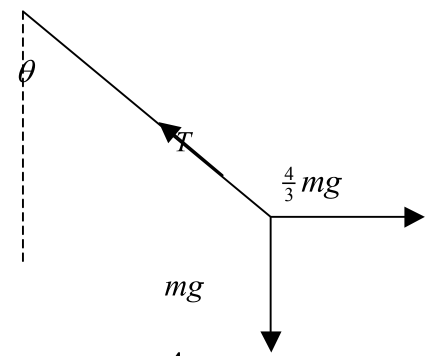
(b) Find the angle that the velocity of the particle makes with the horizontal as it reaches P.

(8)

TOTAL FOR PAPER: 75 MARKS

END

January 2009
6679 Mechanics M3
Mark Scheme

Question Number	Scheme	Marks
1	N2L $\quad 3a = -\left(9 + \frac{15}{(t+1)^2}\right)$	B1
	$3v = -9t + \frac{15}{t+1}(+A)$	M1 A1ft
	$v = 0, t = 4 \quad \Rightarrow \quad 0 = -36 + 3 + A \quad \Rightarrow \quad A = 33$	M1 A1
	$v = -3t + \frac{5}{t+1} + 11$	
	$t = 0 \quad \Rightarrow \quad v = 16$	M1 A1 (7)
		[7]
2	 	
(a)	$(\leftarrow) \quad T\sin\theta = \frac{4}{3}mg$	M1 A1
	$(\uparrow) \quad T\cos\theta = mg$	A1
	$T^2 = \left(\frac{4}{3}mg\right)^2 + (mg)^2$	M1
	Leading to $\quad T = \frac{5}{3}mg$	A1 (5)
(b)	HL $\quad T = \frac{\lambda x}{a} \quad \Rightarrow \quad \frac{5}{3}mg = \frac{3mge}{a}$ ft their T	M1 A1ft
	$e = \frac{5}{9}a$	
	$E = \frac{\lambda x^2}{2a} = \frac{3mg}{2a} \times \left(\frac{5}{9}a\right)^2 = \frac{25}{54}mga$	M1 A1 (4)
		[9]

Question Number	Scheme	Marks
3	$\omega = \dfrac{80 \times 2\pi}{60}$ rad s^{-1} $\left(= \dfrac{8\pi}{3} \approx 8.377... \right)$	B1
	Accept $v = \dfrac{16\pi}{75} \approx 0.67\,\text{ms}^{-1}$ as equivalent	
	$(\uparrow)$ $R = mg$	B1
	For least value of μ $(\leftarrow)$ $\mu mg = mr\omega^2$	M1 A1=A1
	$\mu = \dfrac{0.08}{9.8} \times \left(\dfrac{8\pi}{3} \right)^2 \approx 0.57$ accept 0.573	M1 A1 (7)
		[7]
4 (a)	$a = 8$	B1
	$T = \dfrac{25}{2} = \dfrac{2\pi}{\omega} \Rightarrow \omega = \dfrac{4\pi}{25} (\approx 0.502\ ...)$	M1 A1
	$v^2 = \omega^2 \left(a^2 - x^2 \right) \Rightarrow v^2 = \left(\dfrac{4\pi}{25} \right)^2 \left(8^2 - 3^2 \right)$ ft their $a,\ \omega$	M1 A1ft
	$v = \dfrac{4\pi}{25} \sqrt{55} \approx 3.7 \quad \left(\text{m}\,\text{h}^{-1} \right)$ awrt 3.7	M1 A1 (7)
(b)	$x = a \cos \omega t \Rightarrow 3 = 8 \cos \left(\dfrac{4\pi}{25} t \right)$ ft their $a,\ \omega$	M1 A1ft
	$t \approx 2.3602\ \ldots$	M1
	time is 12 22	A1 (4)
		[11]

Question Number	Scheme	Marks
5 (a)	Let x be the distance from the initial position of B to C GPE lost = EPE gained	
	$mgx\sin 30° = \dfrac{6mgx^2}{2a}$	M1 A1=A1
	Leading to $x = \dfrac{a}{6}$	M1
	$AC = \dfrac{7a}{6}$	A1 (5)
(b)	The greatest speed is attained when the acceleration of B is zero, that is where the forces on B are equal.	
	$(\nwarrow)$ $T = mg\sin 30° = \dfrac{6mge}{a}$	M1
	$e = \dfrac{a}{12}$	A1
	CE $\dfrac{1}{2}mv^2 + \dfrac{6mg}{2a}\left(\dfrac{a}{12}\right)^2 = mg\dfrac{a}{12}\sin 30°$	M1 A1=A1
	Leading to $v = \sqrt{\left(\dfrac{ga}{24}\right)} = \dfrac{\sqrt{6ga}}{12}$	M1 A1 (7)
		[12]
	Alternative approaches to (b) *are considered on the next page.*	

Question Number	Scheme	Marks
5	*Alternative approach to* (b) *using calculus with energy.*	
	Let distance moved by B be x	
	CE $\frac{1}{2}mv^2 + \frac{6mg}{2a}x^2 = mgx\sin 30°$	M1 A1=A1
	$v^2 = gx - \frac{6g}{a}x^2$	
	For maximum v $\frac{\mathrm{d}}{\mathrm{d}x}\left(v^2\right) = 2v\frac{\mathrm{d}v}{\mathrm{d}x} = g - \frac{12g}{a}x = 0$	M1 A1
	$x = \frac{a}{12}$	
	$v^2 = g\left(\frac{a}{12}\right) - \frac{6g}{a}\left(\frac{a}{12}\right)^2 = \frac{ga}{24}$	M1
	$v = \sqrt{\left(\frac{ga}{24}\right)}$	A1 (7)
	Alternative approach to (b) *using calculus with Newton's second law.*	
	As before, the centre of the oscillation is when extension is $\frac{a}{12}$	M1 A1
	N2L $mg\sin 30° - T = m\ddot{x}$	
	$\frac{1}{2}mg - \frac{6mg\left(\frac{a}{12}+x\right)}{a} = m\ddot{x}$	M1 A1
	$\ddot{x} = -\frac{6g}{a}x \quad \Rightarrow \quad \omega^2 = \frac{6g}{a}$	A1
	$v_{\max} = \omega a = \sqrt{\left(\frac{6g}{a}\right)} \times \frac{a}{12} = \sqrt{\left(\frac{ga}{24}\right)}$	M1 A1 (7)

Question Number	Scheme	Marks
6 (a)	$\int y^2\,\mathrm{d}x = \int \left(4-x^2\right)^2 \mathrm{d}x = \int \left(16 - 8x^2 + x^4\right)\mathrm{d}x$	
	$= 16x - \dfrac{8x^3}{3} + \dfrac{x^5}{5}$	M1 A1
	$\left[16x - \dfrac{8x^3}{3} + \dfrac{x^5}{5}\right]_0^2 = \dfrac{256}{15}$	M1 A1
	$\int xy^2\,\mathrm{d}x = \int x\left(4-x^2\right)^2 \mathrm{d}x = \int \left(16x - 8x^3 + x^5\right)\mathrm{d}x$	
	$= 8x^2 - 2x^4 + \dfrac{x^6}{6}$	M1 A1
	$\left[8x^2 - 2x^4 + \dfrac{x^6}{6}\right]_0^2 = \dfrac{32}{3}$	M1A1
	$\bar{x} = \dfrac{\int xy^2\,\mathrm{d}x}{\int y^2\,\mathrm{d}x} = \dfrac{32}{3} \times \dfrac{15}{216} = \dfrac{5}{8}$ *	M1 A1 (10)
(b)	$A \times \bar{x} = \left(\pi r^2 l\right) \times \dfrac{l}{2}$	M1
	$\dfrac{256}{15}\pi \times \dfrac{5}{8} = \pi \times 16l \times \dfrac{l}{2}$	A1 ft
	Leading to $l = \dfrac{2\sqrt{3}}{3}$ accept exact equivalents or awrt 1.15	M1 A1 (4)
		[14]

Question Number	Scheme	Marks
7 (a)	Let speed at C be u	
	CE $\quad \frac{1}{2}mu^2 - \frac{1}{2}m\left(\frac{ag}{4}\right) = mga\left(1-\cos\theta\right)$	M1 A1
	$u^2 = \frac{9ga}{4} - 2ga\cos\theta$	
	$mg\cos\theta \ (+R) = \frac{mu^2}{a}$	M1 A1
	$mg\cos\theta = \frac{9mg}{4} - 2mg\cos\theta$ eliminating u	M1
	Leading to $\cos\theta = \frac{3}{4}$ ∗	M1 A1 (7)
(b)	At C $\quad u^2 = \frac{9ga}{4} - 2ga \times \frac{3}{4} = \frac{3}{4}ga$	B1
	$(\rightarrow)$ $\quad u_x = u\cos\theta = \sqrt{\left(\frac{3ga}{4}\right)} \times \frac{3}{4} = \sqrt{\left(\frac{27ga}{64}\right)} = 2.033\sqrt{a}$	M1 A1ft
	$(\downarrow)$ $\quad u_y = u\sin\theta = \sqrt{\left(\frac{3ga}{4}\right)} \times \frac{\sqrt{7}}{4} = \sqrt{\left(\frac{21ga}{64}\right)} = 1.792\sqrt{a}$	M1
	$v_y^2 = u_y^2 + 2gh \quad \Rightarrow \quad v_y^2 = \frac{21}{64}ga + 2g \times \frac{7}{4}a = \frac{245}{64}ga$	M1 A1
	$\tan\psi = \frac{v_y}{u_x} = \sqrt{\left(\frac{245}{27}\right)} \approx 3.012\ldots$	M1
	$\psi \approx 72°$ awrt $72°$	A1 (8)
	Or 1.3^c (1.2502^c) awrt 1.3^c	**[15]**
	Alternative for the last five marks	
	Let speed at P be v.	
	CE $\quad \frac{1}{2}mv^2 - \frac{1}{2}m\left(\frac{ag}{4}\right) = mg \times 2a$ or equivalent	M1
	$v^2 = \frac{17mga}{4}$	M1 A1
	$\cos\psi = \frac{u_x}{v} = \sqrt{\left(\frac{27}{64} \times \frac{4}{17}\right)} = \sqrt{\left(\frac{27}{272}\right)} \approx 0.315$	M1
	$\psi \approx 72°$ awrt $72°$	A1
	Note: The time of flight from C to P is $\frac{\sqrt{235} - \sqrt{21}}{8}\sqrt{\left(\frac{a}{g}\right)} \approx 1.38373\sqrt{\left(\frac{a}{g}\right)}$	